Quantitative Analysis for Management

Linear Programming Models:
Linear Programming Problem

♦ 1. Tujuan adalah maximize or minimize variabel dependen dari beberapa kuantitas variabel independen (fungsi tujuan).

♦ 2. Batasan-batasan yang diperlukan guna mencapai tujuan.

♦ Tujuan dan Batasan dinyatakan dalam persamaan linear.
Basic Assumptions of Linear Programming

♦ Certainty
♦ Proportionality
♦ Additivity
♦ Divisibility
♦ Nonnegativity
## Flair Furniture Company
### Data - Table 7.1

**Hours Required to Produce One Unit**

<table>
<thead>
<tr>
<th>Department</th>
<th>$X_1$ Tables</th>
<th>$X_2$ Chairs</th>
<th>Available Hours This Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carpentry</td>
<td>4</td>
<td>3</td>
<td>240</td>
</tr>
<tr>
<td>Painting/Varnishing</td>
<td>2</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>Profit/unit</td>
<td>$7$</td>
<td>$5$</td>
<td></td>
</tr>
</tbody>
</table>
Flair Furniture Company
Data - Table 7.1

STEP 1:
Objective: Maximize: $7X_1 + 5X_2$

STEP 2:
Constraints: 
4 $X_1 + 3 X_2 \leq 240$ (carpentry) 
2 $X_1 + X_2 \leq 100$ (painting & varnishing)
STEP 3: Plot Constraints

Flair Furniture Company
Feasible Region

Number of Chairs

Number of Tables

Painting/Varnishing

Carpentry

Feasible Region
Flair Furniture Company Isoprofit Lines

STEP 4: Plot Objective Function

Painting/Varnishing

7X₁ + 5X₂ = 210

Carpentry

7X₁ + 5X₂ = 420

Number of Chairs

Number of Tables
Flair Furniture Company

Optimal Solution

Corner Points

Painting/Varnishing

Carpentry

Solution

\((X_1 = 30, X_2 = 40)\)

Number of Chairs

Number of Tables

120
100
80
60
40
20
0

20
40
60
80
100
Test Corner Point Solutions

Point 1) (0,0) => 7(0) + 5(0) = $0

Point 2) (0,100) => 7(0) + 5(80) = $400

Point 3) (30,40) => 7(30) + 5(40) = $410

Point 4) (50,0) => 7(50) + 5(0) = $350
Solve Equations Simultaneously

To get X1 & X2 values for Point 3:

\[
\begin{align*}
4X1 + 3X2 &\leq 240 \\
X1 &\geq 60 - \frac{3}{4} X2
\end{align*}
\]

\[
\begin{align*}
2X1 + 1X2 &\leq 100 \\
X1 &\geq 50 - \frac{1}{2} X2
\end{align*}
\]

\[
\begin{align*}
60 - \frac{3}{4} X2 &= 50 - \frac{1}{2} X2 \\
60 - 50 &= \frac{3}{4} X2 - \frac{1}{2} X2 \\
10 &= \frac{1}{4} X2 \\
40 &= X2;
\end{align*}
\]

so, \(4X1 + 3(40) = 240\)

\[
\begin{align*}
4X1 &= 240 - 120 \\
X1 &= 30
\end{align*}
\]
Special Cases in LP

- Infeasibility
- Unbounded Solutions
- Redundancy
- Degeneracy
- More Than One Optimal Solution
A Problem with No Feasible Solution

Region Satisfying First 2 Constraints

Region Satisfying 3rd Constraint
A Solution Region That is Unbounded to the Right

Feasible Region

\[ X_1 \geq 5 \]

\[ X_2 \leq 10 \]

\[ X_1 + 2X_2 \geq 10 \]
A Problem with a Redundant Constraint

Feasible Region

\[ 2X_1 + X_2 \leq 30 \]

\[ X_1 + X_2 \leq 20 \]

Redundant Constraint

\[ X_1 \leq 25 \]
An Example of Alternate Optimal Solutions

Optimal Solution Consists of All Combinations of $X_1$ and $X_2$ Along the $AB$ Segment

Isoprofit Line for $8$

Isoprofit Line for $12$

Overlays Line Segment
## Marketing Applications

### Media Selection - Win Big Gambling Club

<table>
<thead>
<tr>
<th>Medium</th>
<th>Audience Reached Per Ad</th>
<th>Cost Per Ad($)</th>
<th>Maximum Ads Per Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV spot (1 minute)</td>
<td>5,000</td>
<td>800</td>
<td>12</td>
</tr>
<tr>
<td>Daily newspaper (full-page ad)</td>
<td>8,500</td>
<td>925</td>
<td>5</td>
</tr>
<tr>
<td>Radio spot (30 seconds, prime time)</td>
<td>2,400</td>
<td>290</td>
<td>25</td>
</tr>
<tr>
<td>Radio spot (1 minute, afternoon)</td>
<td>2,800</td>
<td>380</td>
<td>20</td>
</tr>
</tbody>
</table>

To accompany *Quantitative Analysis for Management, 7e* by Render/Stair

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Win Big Gambling Club

Maximize: \[ 5000X_1 + 8500X_2 + 2400X_3 + 2800X_4 \]

Subject to:

\[ X_1 \leq 12 \text{ (max TV spots/week)} \]
\[ X_2 \leq 5 \text{ (max newspaper ads/week)} \]
\[ X_3 \leq 25 \text{ (max 30-sec. radio spots/week)} \]
\[ X_4 \leq 20 \text{ (max 1-min. radio spots/week)} \]

\[ 800X_1 + 925X_2 + 290X_3 + 380X_4 \leq 8000 \text{ (weekly ad budget)} \]

\[ X_3 + X_4 \geq 5 \text{ (min radio spots/week)} \]

\[ 290X_3 + 380X_4 \leq 1800 \text{ (max radio expense)} \]
## Manufacturing Applications

### Production Mix - Fifth Avenue

<table>
<thead>
<tr>
<th>Variety of Tie</th>
<th>Selling Price per Tie ($)</th>
<th>Monthly Contract Minimum</th>
<th>Monthly Demand</th>
<th>Material Required per Tie (Yds)</th>
<th>Material Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>All silk</td>
<td>6.70</td>
<td>6000</td>
<td>7000</td>
<td>0.125</td>
<td>100% silk</td>
</tr>
<tr>
<td>All polyester</td>
<td>3.55</td>
<td>10000</td>
<td>14000</td>
<td>0.08</td>
<td>100% polyester</td>
</tr>
<tr>
<td>Poly-cotton blend 1</td>
<td>4.31</td>
<td>13000</td>
<td>16000</td>
<td>0.10</td>
<td>50% poly/50% cotton</td>
</tr>
<tr>
<td>Poly-cotton - blend 2</td>
<td>4.81</td>
<td>6000</td>
<td>8500</td>
<td>0.10</td>
<td>30% poly/70% cotton</td>
</tr>
</tbody>
</table>
Maximize: $4.08X_1 + 3.07X_2 + 3.56X_3 + 4.00X_4$

Subject to:

$0.125X_1 \leq 800$ (yards of silk)

$0.08X_2 + 0.05X_3 + 0.03X_4 \leq 3000$ (yards polyester)

$0.05X_3 + 0.07X_4 \leq 1600$ (yards cotton)

$X_1 \geq 6000$ (contract min, silk) $X_1 \leq 7000$ (contract max, silk)

$X_2 \geq 1000$ (contract min, all polyester)

$X_2 \leq 14000$ (contract max, all polyester)

$X_3 \geq 13000$ (contract min, blend1) $X_3 \leq 16000$ (contract max, blend1)

$X_4 \geq 6000$ (contract min, blend2) $X_4 \leq 8500$ (contract max, blend2)
## Manufacturing Applications

### Truck Loading - Goodman Shipping

<table>
<thead>
<tr>
<th>Item</th>
<th>Value ($)</th>
<th>Weight (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22,500</td>
<td>7,500</td>
</tr>
<tr>
<td>2</td>
<td>24,000</td>
<td>7,500</td>
</tr>
<tr>
<td>3</td>
<td>8,000</td>
<td>3,000</td>
</tr>
<tr>
<td>4</td>
<td>9,500</td>
<td>3,500</td>
</tr>
<tr>
<td>5</td>
<td>11,500</td>
<td>4,000</td>
</tr>
<tr>
<td>6</td>
<td>9,750</td>
<td>3,500</td>
</tr>
</tbody>
</table>
Goodman Shipping

Maximize load value: 

\[ 22500 \ X_1 + 24000 \ X_2 + 8000 \ X_3 + 9500 \ X_4 \\
+ 11500 \ X_5 + 9750 \ X_6 \]

Subject to:

\[ 7500 \ X_1 + 7500 \ X_2 + 3000 \ X_3 + 3500 \ X_4 + 4000 \ X_5 \\
+ 3500 \ X_6 \leq 10000 \quad \text{(Capacity)} \]

\[ X_1 \leq 1 \]
\[ X_2 \leq 1 \]
\[ X_3 \leq 1 \]
\[ X_4 \leq 1 \]
\[ X_5 \leq 1 \]
\[ X_6 \leq 1 \]
Flair Furniture Company

Hours Required to Produce One Unit

<table>
<thead>
<tr>
<th>Department</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>Available Hours This Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carpentry</td>
<td>4</td>
<td>3</td>
<td>240</td>
</tr>
<tr>
<td>Painting/Varnishing</td>
<td>2</td>
<td>1</td>
<td>100</td>
</tr>
</tbody>
</table>

Profit/unit

- Tables: $7
- Chairs: $5

Constraints:

- \( 4X_1 + 3X_2 \leq 240 \) (carpentry)
- \( 2X_1 + 1X_2 \leq 100 \) (painting & varnishing)

Objective: Maximize

\( 7X_1 + 5X_2 \)
Flair Furniture Company's Feasible Region & Corner Points

Feasible Region

\[ 4X_1 + 3X_2 \leq 240 \]
\[ 2X_1 + X_1 \leq 100 \]

Corner Points:

- B = (0,80)
- C = (30,40)
- D = (50,0)
Flair Furniture - Adding Slack Variables

**Constraints:**

\[4X_1 + 3X_2 \leq 240\] (carpentry)
\[2X_1 + X_2 \leq 100\] (painting & varnishing)

**Constraints with Slack Variables**

\[4X_1 + 3X_2 + S_1 = 240\] (carpentry)
\[2X_1 + X_2 + S_2 = 100\] (painting & varnishing)

**Objective Function**

\[7X_1 + 5X_2\]

**Objective Function with Slack Variables**

\[7X_1 + 5X_2 + 1S_1 + 1S_2\]
Flair Furniture’s Initial Simplex Tableau

<table>
<thead>
<tr>
<th>Profit per Unit</th>
<th>Production Mix Column</th>
<th>Real Variables Columns</th>
<th>Slack Variables Columns</th>
<th>Constant Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_j$</td>
<td>$X_1$</td>
<td>$X_2$</td>
<td>$S_1$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>$\text{Solution Mix}$</td>
<td>$2$</td>
<td>$1$</td>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\text{Mix}$</td>
<td>$4$</td>
<td>$3$</td>
<td>$0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$Z_j$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$C_j - Z_j$</td>
<td>$7$</td>
<td>$5$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Flair Furniture's Initial Simplex Tableau

**Profit per unit row**

**Constraint equation rows**

**Gross profit row**

**Net profit row**

To accompany *Quantitative Analysis for Management, 7e* by Render/Stair
Pivot Row, Pivot Number Identified in the Initial Simplex Tableau

<table>
<thead>
<tr>
<th>$C_j$</th>
<th>$\begin{array}{c} \text{Solution} \ \text{Mix} \end{array}$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$\text{Quantity}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$S_1$</td>
<td>$2$</td>
<td>$1$</td>
<td>$1$</td>
<td>$0$</td>
<td>$100$</td>
</tr>
<tr>
<td>$0$</td>
<td>$S_2$</td>
<td>$4$</td>
<td>$3$</td>
<td>$0$</td>
<td>$1$</td>
<td>$240$</td>
</tr>
</tbody>
</table>

**Pivot row**

**Pivot number**

**Pivot column**
## Completed Second Simplex Tableau for Flair Furniture

<table>
<thead>
<tr>
<th>$C_j$</th>
<th>$Solution$</th>
<th>$Mix$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$Quantity$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7$</td>
<td>$X_1$</td>
<td>1</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>$0$</td>
<td>$S_2$</td>
<td>0</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td></td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Z_j$</th>
<th>$C_j - Z_j$</th>
<th>$C_j$</th>
<th>$Z_j$</th>
<th>$C_j - Z_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7$</td>
<td>$0$</td>
<td>$0$</td>
<td>$7$</td>
<td>$0$</td>
</tr>
<tr>
<td>$7/2$</td>
<td>$3/2$</td>
<td>$3/2$</td>
<td>$7/2$</td>
<td>$-7/2$</td>
</tr>
</tbody>
</table>

$\text{Z}_j$ values:
- $C_j$ - $Z_j$ for $X_1$: $0$
- $C_j$ - $Z_j$ for $X_2$: $3/2$
- $C_j$ - $Z_j$ for $S_1$: $-7/2$
- $C_j$ - $Z_j$ for $S_2$: $0$

Objective function:
- $Z_j = $350
### Pivot Row, Column, and Number Identified in Second Simplex Tableau

<table>
<thead>
<tr>
<th>$C_j$</th>
<th>$\text{Solution Mix}$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{Mix}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$7$</td>
<td>$X_1$</td>
<td>1</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>$0$</td>
<td>$S_2$</td>
<td>0</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>40</td>
</tr>
</tbody>
</table>

**Pivot row**

<table>
<thead>
<tr>
<th>$Z_j$</th>
<th>$C_j - Z_j$</th>
<th>$\text{Mix}$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>(Total Profit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7$</td>
<td>$0$</td>
<td>$\text{Mix}$</td>
<td>$\text{Mix}$</td>
<td>$X_1$</td>
<td>$X_2$</td>
<td>$S_1$</td>
<td>$S_2$</td>
</tr>
<tr>
<td></td>
<td>$3/2$</td>
<td>$\text{Mix}$</td>
<td>$\text{Mix}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Pivot column**

To accompany *Quantitative Analysis for Management, 7e* by Render/Stair
Calculating the New $X_1$ Row for Flair’s Third Tableau

\[
\begin{align*}
\text{Number in new } X_1 \text{ row} &= \text{Number in old } X_1 \text{ row} - \left[ \text{Number above pivot number} \right] \times \text{Corresponding number in new } X_2 \text{ row} \\
1 &= 1 - (1/2) \times (0) \\
0 &= 1/2 - (1/2) \times (1) \\
3/2 &= 1/2 - (1/2) \times (-2) \\
-1/2 &= 0 - (1/2) \times (1) \\
30 &= 50 - (1/2) \times (40)
\end{align*}
\]
## Final Simplex Tableau for the Flair Furniture Problem

<table>
<thead>
<tr>
<th>$C_j$</th>
<th>$\text{Solution}$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$\text{Quantity}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_j$</td>
<td>Mix</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Z_j$</td>
</tr>
<tr>
<td>$C_j - Z_j$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\text{Quantity}$</td>
</tr>
<tr>
<td>$7$</td>
<td>$X_1$</td>
<td>1</td>
<td>0</td>
<td>$3/2$</td>
<td>$-1/2$</td>
<td>30</td>
</tr>
<tr>
<td>$5$</td>
<td>$X_2$</td>
<td>0</td>
<td>1</td>
<td>$-2$</td>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>$7$</td>
<td>$\text{Total}$</td>
<td>5</td>
<td>$1/2$</td>
<td>$3/2$</td>
<td></td>
<td>$410$</td>
</tr>
</tbody>
</table>

To accompany *Quantitative Analysis for Management, 7e* by Render/Stair
Simplex Steps for Maximization

1. Choose the variable with the greatest positive $C_j - Z_j$ to enter the solution.

2. Determine the row to be replaced by selecting that one with the smallest (non-negative) quantity-to-pivot-column ratio.

3. Calculate the new values for the pivot row.

4. Calculate the new values for the other row(s).

5. Calculate the $C_j$ and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ values greater than zero, return to Step 1.
Surplus & Artificial Variables

Constraints

\[ 5X_1 + 10X_2 + 8X_3 \geq 210 \]
\[ 25X_1 + 30X_2 = 900 \]

Constraints - Surplus & Artificial Variables

\[ 5X_1 + 10X_2 + 8X_3 - S_1 + A_1 = 210 \]
\[ 25X_1 + 30X_2 + A_2 = 900 \]

Objective Function

Min: \[ 5X_1 + 9X_2 + 7X_3 \]

Objective Function - Surplus & Artificial Variables

Min: \[ 5X_1 + 9X_2 + 7X_3 + 0S_1 + MA_1 + MA_2 \]
Simplex Steps for Minimization

1. Choose the variable with the greatest negative $C_j - Z_j$ to enter the solution.

2. Determine the row to be replaced by selecting that one with the smallest (non-negative) quantity-to-pivot-column ratio.

3. Calculate the new values for the pivot row.

4. Calculate the new values for the other row(s).

5. Calculate the $C_j$ and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ values less than zero, return to Step 1.
## Special Cases

### Infeasibility

<table>
<thead>
<tr>
<th>$C_j$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>Qty</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>-2</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>-2</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>M</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>

### Objective Function

- $Z_j = 5X_1 + 8X_2 - 2S_1 + 3M - 21M = 1800 + 20M$

### Comparison

- $C_j - Z_j = 0$
### Special Cases

#### Unboundedness

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>6</th>
<th>9</th>
<th>0</th>
<th>0</th>
<th>Solution Mix</th>
<th>Qty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$X_1$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$X_2$ -1</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$S_2$ -2</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$S_1$ 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$S_2$ 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Qty 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Z_j$ -9</td>
<td>270</td>
</tr>
<tr>
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**Pivot Column**
### Special Cases

#### Degeneracy

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#### Pivot Column

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## Special Cases
### Multiple Optima

<table>
<thead>
<tr>
<th>$C_j$</th>
<th>$Z_j$</th>
<th>$C_j - Z_j$</th>
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<th>$X_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>Qty</th>
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</table>

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Sensitivity Analysis
High Note Sound Company

Max : $50 \, X_1 \, + 120 \, X_2$

Subject to :

$2 \, X_1 \, + 4 \, X_2 \, \leq \, 80$

$3 \, X_1 \, + 1 \, X_2 \, \leq \, 60$
Sensitivity Analysis
High Note Sound Company

Optimal Solution at Point $a$

- $X_1 = 0$ CD Players
- $X_2 = 20$ Receivers
- Profits = $2,400$

Graphical representation:

- $a = (0, 20)$
- $b = (16, 12)$
- $c = (20, 0)$

Iso-Profit Line: $2,400 = 50X_1 + 120X_2$
## Simplex Solution

### High Note Sound Company

<table>
<thead>
<tr>
<th>C&lt;sub&gt;j&lt;/sub&gt;</th>
<th>Solution Mix</th>
<th>X&lt;sub&gt;1&lt;/sub&gt;</th>
<th>X&lt;sub&gt;2&lt;/sub&gt;</th>
<th>S&lt;sub&gt;1&lt;/sub&gt;</th>
<th>S&lt;sub&gt;2&lt;/sub&gt;</th>
<th>Qty</th>
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<tr>
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<td>5/2</td>
<td>0</td>
<td>-1/4</td>
<td>1</td>
<td>40</td>
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<tr>
<td></td>
<td>Z&lt;sub&gt;j&lt;/sub&gt;</td>
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<td>120</td>
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<td>C&lt;sub&gt;j-Zj&lt;/sub&gt;</td>
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### Simplex Solution
#### High Note Sound Company

<table>
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<tr>
<th>C&lt;sub&gt;j&lt;/sub&gt;</th>
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<tr>
<td>C&lt;sub&gt;j&lt;/sub&gt;-Z&lt;sub&gt;j&lt;/sub&gt;</td>
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### Nonbasic Objective Function Coefficients

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<td>120 X&lt;sub&gt;2&lt;/sub&gt;</td>
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<td>1/4</td>
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<td>120</td>
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<tr>
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### Basic Objective Function Coefficients

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<th>$C_j - Z_j$</th>
<th>$Z_j$</th>
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<th>$S_1$</th>
<th>$X_2$</th>
<th>$X_1$</th>
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<td>60+1/2$\Delta$</td>
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<td>0</td>
<td>1/2</td>
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<td>20</td>
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<tr>
<td>120</td>
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<td>120+$\Delta$</td>
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<td>-1/4</td>
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<td>40</td>
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<tr>
<td>0</td>
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<td>30+1/4$\Delta$</td>
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<td>0</td>
<td>0</td>
<td>2400+20$\Delta$</td>
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*To accompany Quantitative Analysis for Management, 7e by Render/Stair*
## Simplex Solution
### High Note Sound Company

<table>
<thead>
<tr>
<th>C_j</th>
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<th>0</th>
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<th>X_2</th>
<th>S_1</th>
<th>S_2</th>
<th>Qty</th>
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<tbody>
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<td>30</td>
<td>0</td>
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<td>-30</td>
<td>0</td>
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</table>

Objective increases by 30 if 1 additional hour of electricians time is available.
Steps to Form the Dual

To form the Dual:

- If the primal is max., the dual is min., and vice versa.
- The right-hand-side values of the primal constraints become the objective coefficients of the dual.
- The primal objective function coefficients become the right-hand-side of the dual constraints.
- The transpose of the primal constraint coefficients become the dual constraint coefficients.
- Constraint inequality signs are reversed.
**Primal & Dual**

**Primal:**

Max: \( 50X_1 + 120X_2 \)

Subject to:

\[
\begin{align*}
2X_1 + 4X_2 & \leq 80 \\
3X_1 + 1X_2 & \leq 60
\end{align*}
\]

**Dual**

Min: \( 80U_1 + 60U_2 \)

Subject to:

\[
\begin{align*}
2U_1 + 3U_2 & \geq 50 \\
4U_1 + 1U_2 & \geq 120
\end{align*}
\]
## Comparison of the Primal and Dual Optimal Tableaus

### Primal’s Optimal Solution

<table>
<thead>
<tr>
<th>( C_j )</th>
<th>Solution Mix</th>
<th>Quantity</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
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<td>1/4</td>
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<td>$5$</td>
<td>( S_2 )</td>
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<td>0</td>
<td>-1/4</td>
<td>1</td>
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<tr>
<td>( Z_j )</td>
<td>( Z_j )</td>
<td>( Z_j )</td>
<td>( Z_j )</td>
<td>( Z_j )</td>
<td>( Z_j )</td>
<td>( Z_j )</td>
</tr>
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<td>( C_j - Z_j )</td>
<td>( C_j - Z_j )</td>
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<td>( C_j - Z_j )</td>
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</tbody>
</table>

### Dual’s Optimal Solution

<table>
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<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
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<td>1</td>
<td>1/4</td>
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<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>$5$</td>
<td>( S_1 )</td>
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<td>-1</td>
<td>1/2</td>
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<tr>
<td>( Z_j )</td>
<td>( Z_j )</td>
<td>( Z_j )</td>
<td>( Z_j )</td>
<td>( Z_j )</td>
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</tbody>
</table>

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